Preemption in multi-mode Resource-constrained project scheduling problem

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Abstract
In this paper we present a new mathematical model for the preemptive multi-mode resource-constrained project scheduling problem (P.MRCPSP), in which multiple execution modes are available for each of the activities of the project. We also introduce the preemptive extension of the problem which allows activity splitting. The objective is minimization the makespan.

Keywords: Preemptive, Multi mode, Resource constrained, Makespan

1. Introduction
The resource-constrained project scheduling problem (RCPSP) is to schedule project activities in order to complete a project in the minimum possible time under the presence of precedence and resource constraints. Precedence constraints are defined between activities (i.e., no activity can be started before finishing all its Predecessors). The multi-mode problem (MRCPSP) is a generalized version of the RCPSP, where each activity can be performed in one out of a set of modes, with a specific activity duration and resource requirements. The objective of the MRCPSP is to find a mode and a start time for each activity such that the makespan is minimized and the schedule is feasible with respect to the precedence and resource constraints. In the non preemptive case once started an activity is not interrupted and runs to completion. The preemptive resource-constrained project scheduling problem (PRCPSP) includes the first relaxation and assumes that activities can be pre-empted at any integer time instant and restarted later on at no additional cost [29]. In the preemptive case which is discussed in this paper an
activity can be interrupted any number of times. As this problem is a generalization of the RCPSP, the P-MRCPSP is also NP-hard [1]. The rest of this paper is organized as follow: the literature review about resource-constrained project scheduling problems is given in Section 2. In Section 3 we first describe the problem and then propose a mathematical model for the preemptive multi-mode Resource-constrained project scheduling problem. Finally, conclusion is presented in Section 4.

2. Literature review

The basic RCPSP assumes that an activity cannot be interrupted once it has been started. Bianco et al. [10], Brucker and Knust [7], Debels and Vanhoucke [11], Demeulemeester and Herroelen [12] and Nudtasomboon and Randhawa [21] allow activity preemption at discrete points in time, that is, an activity can be interrupted after each integer unit of its processing time. e. Franck et al. [14] propose a calendar concept for project scheduling which includes preemptive scheduling. A calendar is defined as a binary function that determines for each period whether activity execution is possible or a break occurs during which an activity may not be started or continued.

Alcaraz et al. [1], Bouleimen and Lecocq [6], Hartmann [15], Jarboui et al. [18], Jozefowska et al. [19], Ozdamar [22] and Pesch [23] while Varma et al. [28] discuss a multi-mode problem without nonrenewable resources. Multi-mode problems with generalized precedence constraints have been considered by Barrios et al. [3], Brucker and Knust [7], Calhoun et al. [8], de Reyck and Herroelen [24], Drexel et al. [13], Heilmann [16-17], Nonobe and Ibaraki [20], and Sabzehparvar and Seyed Hosseini [25]. Zhu et al. [32] employ a multi-mode problem with generalized resource constraints. Salewski et al. [26] and Drexel et al. [13] extend the multi-mode RCPSP by introducing socalled mode identity constraints. The motivation for this is that there may be several activities that should be performed in the same way, e.g., by allocating the same resources to them. To cover this, the set of all activities is partitioned into sets of activities $H_u, u = 1, \ldots, U$. The activities of each set $H_u$ must be performed in the same mode. That is, $M_i = M_j$ must hold for all activities $i, j \in H_u$ (note that this requires $M_i = M_j$). Schultmann and Rentz [28] present a case study that demonstrates how the multi-mode RCPSP can be applied to projects which consist of the dismantling of buildings.

Voß and Witt [30] employ the multi-mode RCPSP with an objective that contains makespan, weighted tardiness and setup costs. The inclusion of setup costs supports batching of activities. The problem setting is motivated by a production planning problem at a steel manufacturer. Bomsdorf and Derigs [13] employ an objective for movie shooting projects that consists of several components which are allowed to be squared. The components include specific criteria such as the minimization of location changes over time (each activity is associated with a location). Al-Fawzan and Haouari [2] combine makespan minimization and maximization of total free slack into one objective. Another way to deal with multiple objectives is the generation of Pareto-optimal schedules. This approach is followed by several authors. Davis et al. [9] minimize the makespan as well as the overutilization of each renewable resource.
3. Proposed model

3.1. Problem definition

The project is represented as an activity-on-the-node network \( G (N, A) \), where \( N \) is the set of activities and \( A \) is the set of pairs of activities between which a finish-start precedence relationship with a minimal time lag of 0 exists. A set of activities, numbered from 1 to \( |N| \) with a dummy start node 0 and a dummy end node \( |N| + 1 \), is to be scheduled on a set \( R \) resource. Each activity \( i \in N \) is performed in a mode \( m_i \), which is chosen out of a set of \( |M_i| \) different execution modes \( M_i={1,..., |M_i|} \). The duration of activity \( i \), when executed in mode \( m_i \), is \( d_{im_i} \). Each mode \( m_i \) requires \( r_{im_i,k} \) resource units \((k \in R)\). A schedule \( S \) is defined by a vector of activity start times \( s_i \) and a vector denoting its corresponding execution modes \( m_i \). A schedule is said to be feasible if all precedence and resource constraints are satisfied. The objective of the P-MMRCPS is to minimize the makespan of the project.

3.2. Objective function

The objective is minimization the makespan. The makespan is the completion time of a project that equals the completion time of activity \( n \).

3.3. Mathematical model

In this section, we present a novel mathematical model for a preemptive multi-mode resource constrained project scheduling problem (MRCPS).

3.3.1. Indices and Parameters and Variables

- \( \mathcal{T} \)  Project time window
- \( N \) Number of activity
- \( i \) Index of activity
- 0 Dummy start node
- \( n + 1 \) Dummy end node
- \( m \) Index of mode
- \( S_{i,lm} \) Start time of \( l^{th} \) units of activity \( i \) in mode \( m \) where each activity \( i \) is broken into \( d_{im} \)
- \( d_{im} \) Duration of Activity \( i \) executed mode \( m \)
- \( t \) Index for period of time
- \( k \) Index of nonrenewable resource
- \( a_k \) Availability of each nonrenewable resource type \( k \) in each time period
- \( r_{im,k} \) Each activity \( i \) in mode \( m \) requires \( r_{im,k} \) nonrenewable resource units
A very large positive number

-1 if activity i is completed in mode m

0 otherwise

### 3.3.2. Proposed mathematical model

The MRCPSP can be stated as follows:

\[
\begin{align*}
\min & \quad \text{Makespan} = \min \left\{ S_{\text{FIN}} \right\} \\
\text{S.T.} & \quad S_{\text{FIN}} \leq T \\
& \quad S_{\text{FIN}} + 1 \leq S_{\text{FIN}} + (1 - x_{im})E + (1 - x_{im})E \quad m = 1, M, \quad i = 1, N \\
& \quad S_{\text{FIN}} + 1 \leq S_{\text{FIN}} + (1 - x_{im})E \quad m = 1, M, \quad i = 1, N \\
& \quad \sum_{m=1}^{M} x_{im} = 1 \\
& \quad \sum_{i=1}^{N} \sum_{m=1}^{M} x_{im} = q_j \\
& \quad S_{\text{FIN}} = 0 \\
& \quad S_{\text{FIN}} \in \text{int} \\
\end{align*}
\]

The objective function (1) minimizes the total makespan of the project. In Constraint set (2) makes the makespan not to take more than T. Constraint set (3) the earliest start time of an activity j cannot be smaller than the finish time for the last unit of duration of its predecessor i. E is a very large positive number If activity i,j do not execute in mode m, E causes this Constraint to be ineffective. Constraint set (4) guarantees that the start time for every time instance of an activity has to be at least one time-unit larger than the start time for the previous unit of duration if activity i dose not execute in mode m, E causes this Constraint to be ineffective. Each activity i has to be performed in exactly one mode mi (constraint (5)). Constraints (6) take care of the nonrenewable resource limitations, respectively. Constraint (7) forces the project to start at time instance zero and constraint (8) ensures that the activity start times assume nonnegative integer. A schedule which fulfills all the constraints (1 to 8), is called optimal.

### 4. CONCLUSION

The previous researches have shown that activity preemption drastically increases the problem complexity. In this paper, we have introduced a novel mathematical model for the preemptive multi-mode resource constrained project scheduling problem (P-
MRCPSP). In fact we have extended the resource constrained project schedule problems by considering preemption. The objective was minimization the makespan, Areas of further researches to propose other models are considering new constraints and new objectives for RCPSP. In this paper resources were nonrenewable. Further research can consider renewable resources. Clearly for solving this kind of problems in large size we need to use heuristic algorithms.
References


